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# Phase structure of lattice QCD at finite temperature for 2+1 flavors of Kogut-Susskind quarks *

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We report on a study of the finite-temperature chiral transition on an $N_t = 4$ lattice for $2+1$ flavors of Kogut-Susskind quarks. We find the point of physical quark masses to lie in the region of crossover, in agreement with results of previous studies. Results of a detailed examination of the $m_{u,d} = m_s$ case indicate vanishing of the screening mass of $\sigma$ meson at the end point of the first-order transition.

## 1. Introduction

An important issue in finite-temperature lattice QCD is the determination of the nature of the chiral phase transition for a realistic spectrum of light up and down quarks and a heavier strange quark. Despite its importance, past studies of this "2+1" case have been few. For the Kogut-Susskind action, all of them have been made around 1990[1–3].

It was found in these studies that the chiral phase transition changes from a first-order transition to a crossover as the strange quark mass $m_s$ increases beyond a critical value $m_s^c$ for a fixed degenerate up and down quark mass $m_{u,d}$, in agreement with predictions of an effective $\sigma$ model of QCD[4,5]. Results were also obtained[3,2] which indicate the physical point of quark masses to lie in the region of crossover on the $(m_{u,d}, m_s)$ plane. However, these results were based on simulations made at only a few sets of quark masses. Clearly a more extensive study is called for to have a full understanding of the phase structure in the 2-parameter space of $(m_{u,d}, m_s)$. Here we report first results from our recent effort toward this goal.

An interesting suggestion from a $\sigma$ model analysis[5] is that the second-order transition expected at the critical strange quark mass $m_s^c$ is in the Ising universality class, with the massless mode provided by the $\sigma$ meson. A novel feature of our work is a study of the screening mass of $\sigma$ to examine this point.

Our simulations are performed for the temporal lattice size $N_t = 4$. An $8^3 \times 4$ lattice is employed to make a survey of the phase structure varying $\beta$, $m_{u,d}$ and $m_s$. A detailed investigation is then made along the flavor $SU(3)$ symmetric line $m_{u,d} = m_s$ by another series of simulations on a $16 \times 8^2 \times 4$ and a $16^3 \times 4$ lattice. For each parameter set, (1–2)$\times 10^3$ trajectories of unit length are generated by the hybrid R algorithm.

## 2. Phase diagram on the $(m_{u,d}, m_s)$ plane

We show the result of our phase diagram analysis on an $8^3 \times 4$ lattice in Fig. 1. At $m_{u,d} = m_s = 0.01$ a clear two-state signal is obtained by a comparison of runs with a hot and a cold start.

*Presented by S. Kaya.

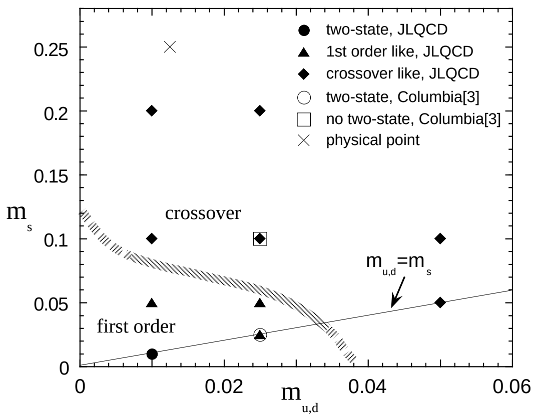


Figure 1. Phase diagram on $(m_{u,d}, m_s)$ plane obtained on a $8^3 \times 4$ lattice. Open symbols are results of Ref. [3] on a $16^3 \times 4$ lattice. Shaded line represents an estimate of the critical line, and cross the physical point(see text).

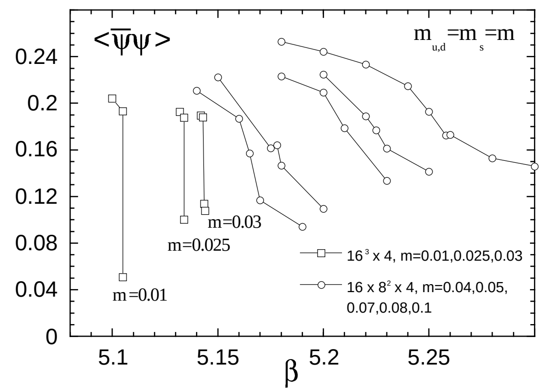


Figure 2. $\langle\overline{\psi}\psi\rangle$ as a function of $\beta$ for various quark masses along the line $m_{u,d} = m_s \equiv m$.

At triangle points, we find a very sharp change of observables over a narrow range of $\beta$, suggestive of a first-order transition, while only a smooth crossover is seen at diamond points.

In Fig. 2 we plot results for the chiral condensate along the line $m_{u,d} = m_s \equiv m$ on a $16^3 \times 4$ lattice ($m \leq 0.03$) or a $16 \times 8^2 \times 4$ lattice ($m \geq 0.04$). On these lattices a two-state signal is found, which is clear at $m = 0.01$ and 0.025, but less so at $m = 0.03$. The behavior above $m = 0.04$, on the other hand, indicates a crossover.

The results taken together suggest that the critical line marking the end point of the first-order transition runs above the triangle points at $m_{u,d} \leq 0.025$, and below the point $m_{u,d} = m_s = 0.04$ in Fig. 1, which is indicated by the shaded line. If we assume $T_c = 150$ MeV for the critical temperature, the physical point is located at the cross in Fig. 1, which is in the region of crossover. Our results are in agreement with those of the Columbia group obtained at $m_{u,d} = 0.025$ on a $16^3 \times 4$ lattice[3].

## 3. Results along the $m_{u,d} = m_s$ line

We now discuss results along the line $m_{u,d} = m_s \equiv m$. Given our observation of a two-state signal at $m = 0.01, 0.025$, one way to estimate the value of the critical mass $m^c$ is to extrapolate the gap of the chiral condensate $\Delta\langle\overline{\psi}\psi\rangle$ toward larger $m$ where it vanishes. Employing the form $\Delta\langle\overline{\psi}\psi\rangle \propto (m^c - m)^{1/2}$ predicted by the mean-field analysis of the $\sigma$ model, we find $m^c \simeq 0.034$. If we use a naive linear extrapolation, we obtain $m^c \simeq 0.049$. A similar value $m^c \approx 0.045$ was previously reported[6] by a linear extrapolation applied to old results[2,3].

In the region of crossover $m > m^c$, we expect the peak height of the chiral susceptibility $\chi_m$ to develop a singular behavior $\chi_m \propto (m - m^c)^{-z}$ as $m \to m^c$. We calculate $\chi_m$ for $m \geq 0.04$ with the histogram reweighting method. Assuming $m^c = 0.034$, we fit the peak height to the form above. A reasonable fit with $\chi^2/df = 1.05$ is obtained with the value of the exponent $z = 0.67(3)$, which is comparable to the Ising value $z \simeq 0.79$ and the mean-field value 2/3.

In order to examine the screening mass $M_\sigma$ of $\sigma$ meson, we employ $U(1)$ random source and no gauge fixing to evaluate the two quark loop contribution of the $\sigma$ propagator. Good results are obtained for the full $\sigma$ propagator with this method as illustrated in Fig. 3.

The quark mass dependence of $\pi$ and $\sigma$ screening masses for $m \leq 0.03$, where we find a first-order transition, is plotted in Fig. 4(a). We observe that $M_\sigma^2$ decreases toward zero as $m$ increases toward the critical value, both on the confined and the deconfined side of the transition, in contrast to $M_\pi^2$ which increases.

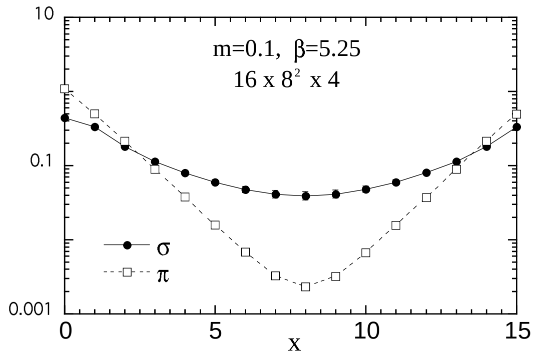


Figure 3. $\pi$ and $\sigma$ propagators for $m = 0.1$ at the transition point $\beta$=5.25.

Assuming a linear quark mass dependence, $M_\sigma^2 \propto m^c - m$, predicted by the mean-field analysis of the $\sigma$ model, we obtain $m^c = 0.034(3)$ in the confining phase and $m^c = 0.031(3)$ in the deconfining phase. These values are consistent with each other, and are also in agreement with the estimate from a square root extrapolation of the gap of $\langle\overline{\psi}\psi\rangle$ discussed above. These results indicate vanishing of the $\sigma$ mass at the critical quark mass as suggested by the $\sigma$ model[5].

Results for larger quark masses ($m \geq 0.04$), where a crossover behavior is seen, is shown in Fig. 4(b). While $M_\sigma^2$ decreases toward smaller values of $m$, the variation is too mild to attempt an independent estimate of $m^c$. An interesting point which requires clarification is that $M_\sigma^2$ stays considerably small compared to $M_\pi^2$ even for large quark masses.

## 4. Conclusions and future work

Our study with the Kogut-Susskind action supports the previous conclusion with this action that there is no finite-temperature phase transition for three flavors of quarks with physical masses. This means that a discrepancy with the conclusion from the Wilson action[7] still remains.

We also find a strong indication that the screening mass of $\sigma$ vanishes at the end point of the first-order transition along the line $m_{u,d} = m_s$.

We plan to extend analyses carried out here to the $m_{u,d} \neq m_s$ case to further explore the real-

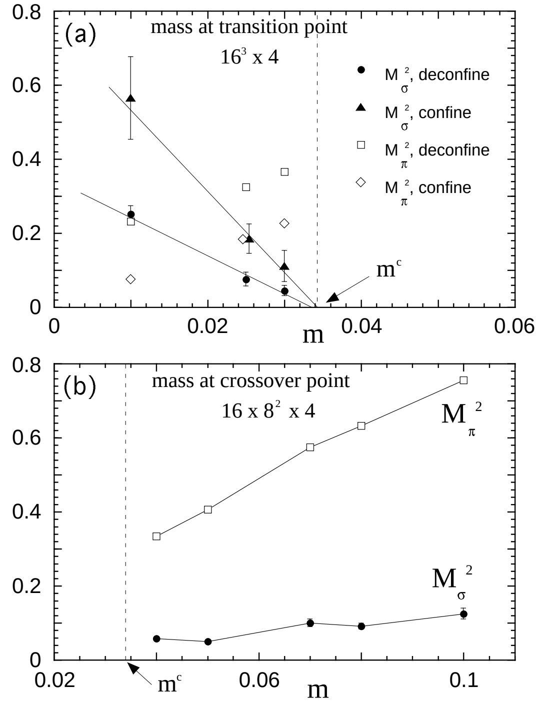


Figure 4. Screening masses $M_\sigma^2$ and $M_\pi^2$ as a function of $m$ for (a) $m \leq 0.03$ and (b) $m \geq 0.04$ at the transition point. In (a) results on the two sides of first-order transition are shown.

world QCD chiral transition.

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